Solution:

a| 5 $\frac{dy(t)}{dt} + 10y(t) = 9x(t)$ zero initial condition.

$\xrightarrow{\mathcal{L}} sY(s) + 10Y(s) = 9X(s)$ $\boxed{3}$

$H(s) = \frac{Y(s)}{X(s)} = \frac{9}{s+10}$ $\boxed{2}$

b| 5 Step response, so $X(s) = \frac{1}{s}$ $\boxed{1}$

$Y(s) = X(s)H(s) = \frac{9}{s(s+10)}$ $\boxed{1}$

$= \frac{0.9}{s} - \frac{0.9}{s+10}$ $\boxed{2}$

$\xrightarrow{\mathcal{L}^{-1}} y(t) = 0.9 - 0.9e^{-10t}$ $\boxed{1}$

c| 5 $X(s) = \frac{\omega}{s^2+\omega^2}$ $\boxed{0.5}$

$Y(s) = X(s)H(s) = \frac{9\omega}{(s^2+\omega^2)(s+10)}$ $\boxed{0.5}$

$= \frac{A}{s+j\omega} + \frac{B}{s-j\omega} + \frac{C}{s+10}$

Using Cauchy's residue theorem:

$A = \frac{j}{2\omega}(j\omega+10) = \frac{\omega+10j}{2\omega}$ $\boxed{1}$

$B = -\frac{j}{2\omega}(j\omega+10) = \frac{\omega-10j}{2\omega}$ $\boxed{1}$

$C = \frac{9\omega}{100+\omega^2}$ $\boxed{1}$

$\xrightarrow{\mathcal{L}^{-1}} y(t) = Ae^{-j\omega t} + Be^{j\omega t} + Ce^{-10t}$

$= (A+B)\cos(\omega t) + j(B-A)\sin(\omega t) + Ce^{-10t}$

$= 4\cos\omega t + \frac{\omega}{10}\sin\omega t + \frac{9\omega}{100+\omega^2}e^{-10t}$ $\boxed{1}$

d| 5 Closed-loop transfer function

$\frac{H(s)}{1+KH(s)} = \frac{9}{s+10+9k}$ $\boxed{3}$

when $k=10$: $= \frac{9}{s+100}$ $\boxed{1}$

pole: $s = -100$ $\boxed{1}$

e| 6 $\tau_p = \frac{1}{10}$ $\boxed{1}$

Impulse response: $X(s) = 1$ $\boxed{1}$

$Y(s) = X(s)H(s) = \frac{9}{s+10}$ $\boxed{1}$

$\xrightarrow{\mathcal{L}^{-1}} y(t) = 9e^{-10t}$

When $t = \tau_p$, percentage of decrease:

$\frac{y(t=0) - y(t-\tau_p)}{y(t=0) - y(t\to\infty)} = \frac{9-9e^{-1}}{9-0}$

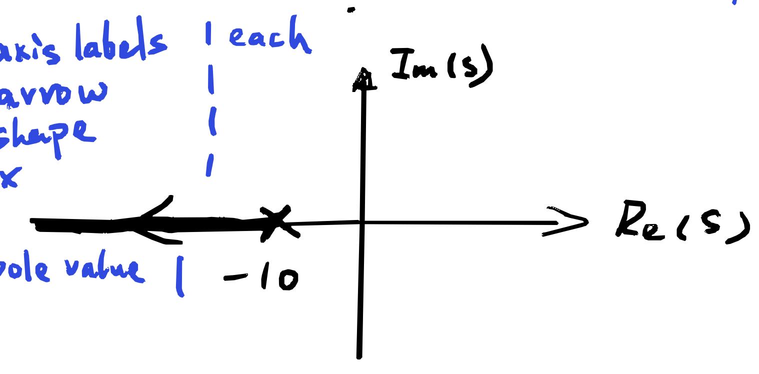
$= 1 - e^{-1} \approx 63.21\%$ $\boxed{2}$

f. 5 Yes, stable. $\boxed{1}$

when $k=100$, closed-loop: $\frac{9}{s+910}$

$s = -910$. $\boxed{2}$

g. 6 axis labels | each
arrow |
shape |
x |
pole value | -10



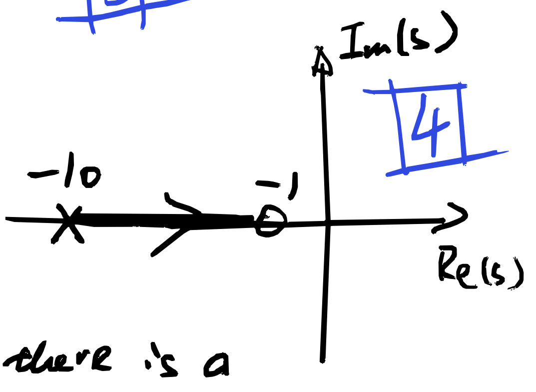
h| 10 $k(1+s)$ as controller

Thus the closed-loop transfer function

$\frac{k(1+s)}{(1+k)s+10+k}$ $\boxed{3}$

Zero: $s = -1$

Pole: $s = -\frac{10+k}{1+k}$



$\boxed{4}$

As k increase, because there is a zero at -1 now, the closed-loop poles becomes closer to the imaginary axis. When k becomes very large, the closed-loop poles will converge to the zero at -1. $\boxed{3}$

i| 12 $k(1+\frac{1}{s}+s)\frac{9}{s+10}$

$= 9k\frac{s^2+s+1}{s^2+10s}$ $\boxed{2}$

Closed-loop: $\frac{9k(s^2+s+1)}{s^2+10s+9k(s^2+s+1)}$

$\boxed{3}$ $= \frac{9k(s^2+s+1)}{(9k+1)s^2+(10+9k)s+9k}$

Based on the final value theorem:

$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$ $\boxed{2}$

Thus, for impulse response:

$\lim_{s\to 0} \frac{s\,9k(s^2+s+1)}{(9k+1)s^2+(10+9k)s+9k} = 0$ $\boxed{5}$

for step response

$\lim_{s\to 0} \frac{9k(s^2+s+1)}{(9k+1)s^2+(10+9k)s+9k} = 1$ any one is okay.

They reaches the steady-state values.

Thus error is zero.

$$H_c(s)H_p(s) = \frac{k(s+1)}{4s^2+64s+32}$$

$$H_c(s) = k,\quad H_p(s) = \frac{s+1}{4s^2+64s+32} = \frac{s+1}{(s+16)(4s+2)}$$

a 4

Zero: -1 ; poles $-\frac{1}{2}, -16$

b See sketch sheet

c 7

1. $$\frac{k(s+1)}{4s^2+(64+k)s+32+k}$$ 3

2. $$\frac{H_c(s)H_p(s)}{1+H_c(s)H_p(s)H_o(s)}$$

$$= \frac{\frac{k(s+1)}{4s^2+64s+32}}{1+\frac{k(s+1)}{4s^2+64s+32}\cdot\frac{s+2}{s-5}}$$

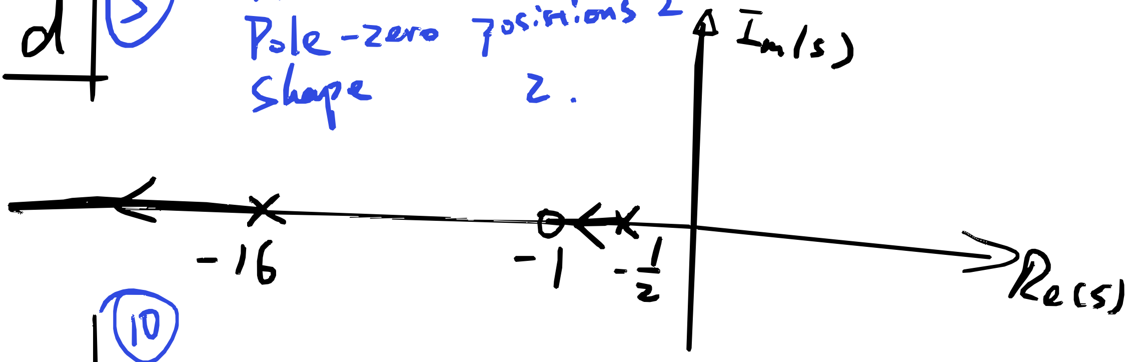
$$= \frac{k(s+1)(s-5)}{(4s^2+64s+32)(s-5)+k(s+1)(s+2)}$$

$$= \frac{k(s^2-4s-5)}{4s^3+64s^2+32s-20s^2-320s-160+k(s^2+3s+2)}$$

$$= \frac{k(s^2-4s-5)}{4s^3+(44+k)s^2+(3k-288)s-160}$$ 4

d 5

Axis 1
Pole-zero positions 2
Shape 2.



e 10

① $Z = N + P$

$N = 0$ 1 $P = 0$ 1 $Z = 0$ 2

Stable 1

② Passes -1, so marginally stable 3 2

f 15 $Z = N + P$ $\qquad P = 1$ 2

If $N=0$, then $Z=1$, $k<80$ 3

If $N=1$, then $Z=2$ $80<k<100$ 3

If $N=-1$, then $Z=0$, $k>100$ 3

Why? Nyquist plot plots: $\frac{k(s+1)}{4s^2+64s+32}\cdot\frac{s+2}{s-5}$ 4 explanation

$$\frac{k(s+1)}{4s^2+64s+32}\cdot\frac{s+2}{s-5}+1$$

$$= \frac{(4s^2+64s+32)(s-5)+k(s+1)(s+2)}{(4s^2+64s+32)(s-5)}$$

closed-loop poles

open-loop poles & 5

So $P = 1$ because $(s-5)$

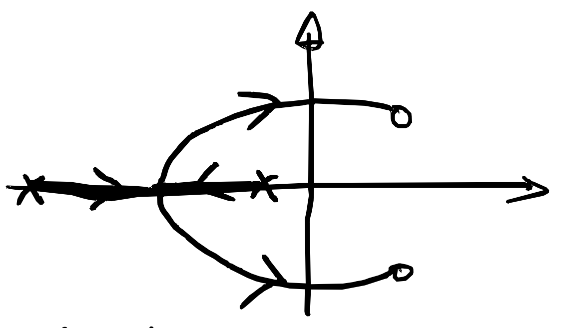
Thus when we have $k<80$, no encirclement then 1 unstable closed-loop pole.

When k is larger, small CW encirclement, $N=1$ then 2 unstable closed-loop poles.

when k is even larger, large CCW encirclement, $N=-1$, then 0 unstable closed-loop poles.

g 10 Openloop: $\frac{s^2-2s+2}{s+1}\cdot\frac{s+1}{(4s+2)(s+16)}$

$$= \frac{(s-1+j)(s-1-j)}{(4s+2)(s+16)}$$ 2



4

Closed-loop:

$$\frac{s^2-2s+2}{4s^2+64s+32+ks^2-2ks+2k}$$

$$= \frac{s^2-2s+2}{(4+k)s^2+(64-2k)s+32+2k}$$ 2

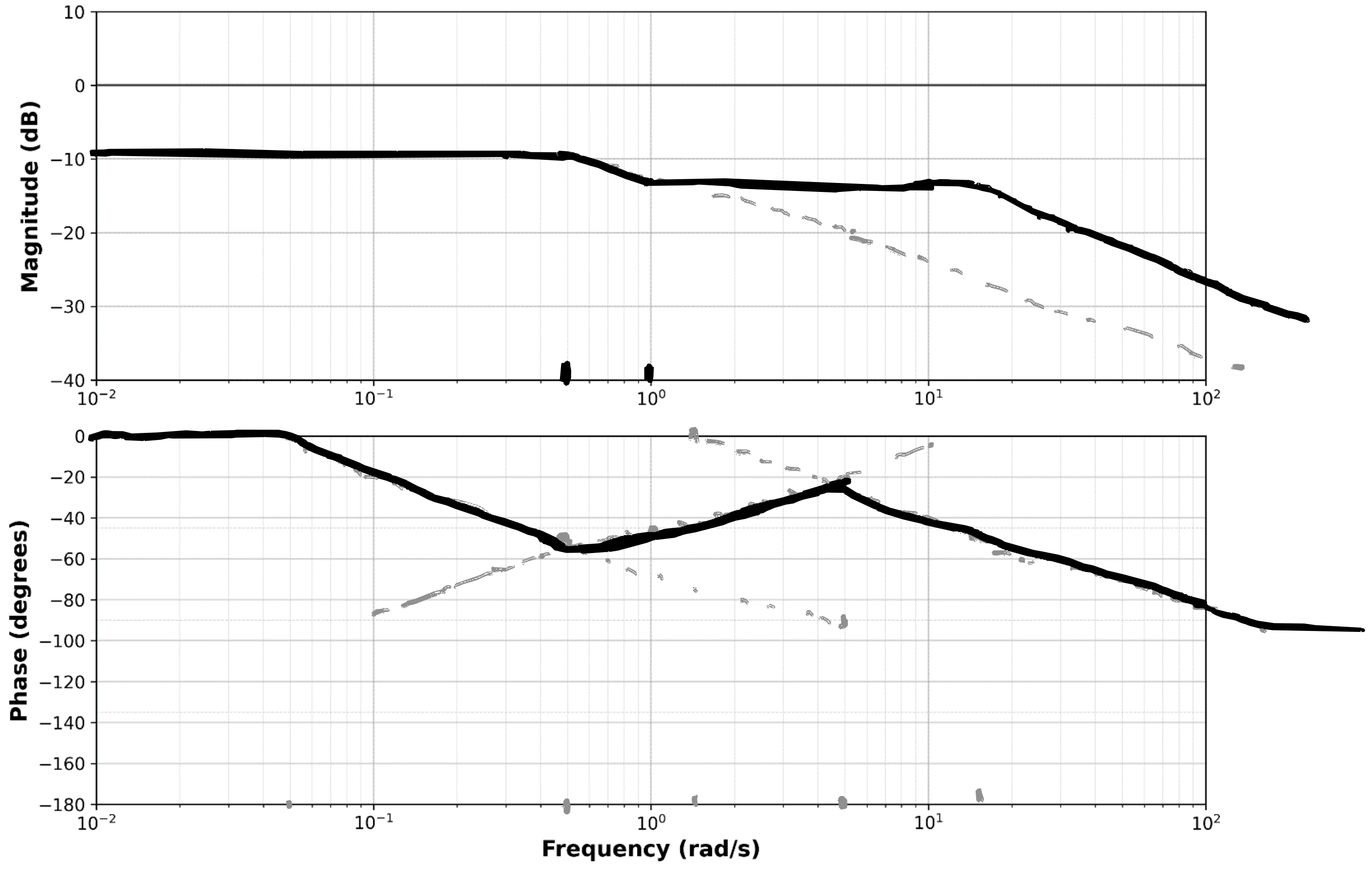
Poles: $$s = \frac{2k-64 \pm \sqrt{(64-2k)^2-4(4+k)(32+2k)}}{8+2k}$$

$2k-64=0 \qquad k=32$ 2

Student name: ______________________

Student number: ______________________

Bode Plot Sketch Sheet



$$\frac{10(s+1)}{(4s+2)(s+16)} \qquad K_{DC} = \frac{10}{32}$$

Corner freq

$1 - 10^0$

$0.5 - 10^{-1} \times 5$

$16 - 1.6 \times 10^1$

$20 \log_{10}\left(\frac{10}{32}\right) \approx -10.1 dB$

Zero: -1

pole: -0.5, -16